A Method for the Reduction of Comet Photographs.

By H. C. Plummer, M.A.

1. The magnificent results which have lately been obtained in comet photography suggest some reflections on the method by which the plates may be reduced. The method of Bessel,* which was designed for the treatment of Halley's comet in 1835, and has been followed more or less faithfully by all subsequent writers, deals with observed position-angles and distances with respect to the nucleus. If the observations are not made in this form, they must first be reduced to it. Of course position-angles and distances can be very easily deduced from the photographic plate.† But it has frequently been found ‡ that there is an advantage in the use of rectangular coordinates over spherical geometry in dealing with photographs. There is a corresponding change in the method of calculation from the general use of logarithms to the calculating machine which appeals strongly to some computers. It may, therefore, be worth while to investigate the formulæ which seem to be the natural expression of this point of view. Whether the emanations from a comet remain strictly in the plane of the orbit is open to doubt, but provisionally it is necessary to make this assumption, and it forms the basis of what follows.

2. It will be convenient to give precepts for certain preliminary operations. Let us form the arrays

$$\begin{vmatrix} -\sin A & \cos A & 0 \\ -\sin D \cos A & -\sin D \sin A & \cos D \\ \cos D \cos A & \cos D \sin A & \sin D \end{vmatrix} \quad . \quad (1)$$

where A, D are the R.A. and declination of the plate-centre, and

$$\begin{vmatrix} X & Y & Z \\ a\sin(\alpha+u) & b\sin(\beta+u) & c\sin(\gamma+u) \\ a\cos(\alpha+u) & b\cos(\beta+u) & c\cos(\gamma+u) \end{vmatrix} \quad . \quad (2)$$

where X, Y, Z are the rectangular coordinates of the Sun referred to the equatorial system of coordinates, $a, b, c, \alpha, \beta, \gamma$ the Gaussian constants for the orbit of the comet, and u an angle measured from the ascending node in the direction of the comet's motion: the actual value to be assigned to u can be left for definition later on. Next we form the array

$$\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \quad . \quad . \quad . \quad . \quad (3)$$

* *A.N.*, No. 300. † Cf. *M.N.*, lxix. p. 100.
‡ Cf. *M.N.*, lx. pp. 176 and 201 ; lxii. p. 29.

which is formed by the products by rows from the arrays (1) and (2), so that

$$
\begin{aligned}
l_1 &= -\sin A . X + \cos A . Y \\
m_1 &= -\sin A . a \sin(\alpha+u) + \cos A . b \sin(\beta+u) \\
n_1 &= -\sin A . a \cos(\alpha+u) + \cos A . b \cos(\beta+u) \\
l_2 &= -\sin D \cos A . X - \sin D \sin A . Y + \cos D . Z \\
m_2 &= -\sin D \cos A . a \sin(\alpha+u) - \sin D \sin A . b \sin(\beta+u) + \cos D . c \sin(\gamma+u) \\
n_2 &= -\sin D \cos A . a \cos(\alpha+u) - \sin D \sin A . b \cos(\beta+u) + \cos D . c \cos(\gamma+u) \\
l_3 &= \cos D \cos A . X + \cos D \sin A . Y + \sin D . Z \\
m_3 &= \cos D \cos A . a \sin(\alpha+u) + \cos D \sin A . b \sin(\beta+u) + \sin D . c \sin(\gamma+u) \\
n_3 &= \cos D \cos A . a \cos(\alpha+u) + \cos D \sin A . b \cos(\beta+u) + \sin D . c \cos(\gamma+u)
\end{aligned}
$$

Lastly we form the conjugate array

$$
\begin{vmatrix} L_1 & M_1 & N_1 \\ L_2 & M_2 & N_2 \\ L_3 & M_3 & N_3 \end{vmatrix} . \quad . \quad . \quad . \quad . \qquad (4)
$$

the constituents of which are the minors

$$
\begin{aligned}
L_1 &= m_2 n_3 - m_3 n_2, \quad M_1 = l_2 n_3 - l_3 n_2, \quad N_1 = l_2 m_3 - l_3 m_2 \\
L_2 &= m_1 n_3 - m_3 n_1, \quad M_2 = l_1 n_3 - l_3 n_1, \quad N_2 = l_1 m_3 - l_3 m_1 \\
L_3 &= m_1 n_2 - m_2 n_1, \quad M_3 = l_1 n_2 - l_2 n_1, \quad N_3 = l_1 m_2 - l_2 m_1
\end{aligned}
$$

The operations involved in computing the constituents of the arrays (3) and (4) will be very easily carried out with the help of an arithmometer.

3. Let ρ, θ be the polar coordinates of a point in the plane of the orbit, the origin being the Sun and the initial line making an angle u with the direction of the node. Then the geocentric coordinates of the point are

$$
\begin{aligned}
x_1 &= a\rho \sin(\alpha + u + \theta) + X \\
&= ax \sin(\alpha+u) + ay \cos(\alpha+u) + X \\
y_1 &= bx \sin(\beta+u) + by \cos(\beta+u) + Y \\
z_1 &= cx \sin(\gamma+u) + cy \cos(\gamma+u) + Z
\end{aligned}
$$

But if x_2, y_2, z_2 are the coordinates of the same point referred to the axes O A, O B, O C, where C is the plate-centre, and the plane O C B passes through the pole P,

$$
\begin{aligned}
x_2 &= -\sin A . x_1 + \cos A . y_1 \\
y_2 &= -\sin D \cos A . x_1 - \sin D \sin A . y_1 + \cos D . z_1 \\
z_2 &= \cos D \cos A . x_1 + \cos D \sin A . y_1 + \sin D . z_1
\end{aligned}
$$

the coefficients being given by the array (1). Hence

$$
\begin{aligned}
x_2 &= l_1 + m_1 x + n_1 y \\
y_2 &= l_2 + m_2 x + n_2 y \\
z_2 &= l_3 + m_3 x + n_3 y
\end{aligned}
$$

But if ξ, η are the standard coordinates of the image of the point on the plate, f is the focal length, and parallax is neglected,

$$
\xi / x_2 = \eta / y_2 = f / z_2
$$

so that x, y, the rectangular coordinates in the plane of the orbit, are connected with ξ, η by the relations

$$(l_1 + m_1x + n_1y)/\xi = (l_2 + m_2x + n_2y)/\eta = (l_3 + m_3x + n_3y)/f \quad (5)$$

which are equivalent to

$$L_1\xi - L_2\eta + L_3f = (-M_1\xi + M_2\eta - M_3f)/x = (N_1\xi - N_2\eta + N_3f)/y \quad (6)$$

Thus the sets of coefficients in (3) or (4) enable us to pass easily from points in the plane of the orbit to the corresponding points on the photograph, or *vice versa*.

4. The value to be assigned to u has purposely been left arbitrary. If, for example, we put $u = 0$, the axis of x will be the line from the Sun to the ascending node of the comet. But if we wish to form a representation of a number of different points connected with the comet at the date of a particular photograph, it is natural to make the radius vector of the nucleus the axis of x, and then we must put

$$u = \varpi - \Omega + v$$

where v is the true anomaly of the nucleus at the date of the photograph. On the other hand, if we wish to follow the actual motion of a particular point as shown by a series of photographs at different times, it will be advantageous to use a fixed axis of x by putting

$$u = \varpi - \Omega + v_0$$

where v_0 is a constant chosen for convenience, and may be the true anomaly of the nucleus at the time of one of the photographs, or may be chosen for some particular reason; *e.g.* if we put $v_0 = 0$, the axis of x will be drawn from the Sun in the direction of perihelion. In any case the Sun is at the origin of coordinates.

5. The projective relation between corresponding points on the photograph and on the plane of the comet's orbit make it easy to transform any curved aggregate of points on the photograph into the corresponding curve in space by means of (5). Thus if we observe a straight streamer whose equation on the plate is

$$A\xi + B\eta + C = 0$$

this corresponds in the plane of the orbit to

$$Al_1 + Bl_2 + Cl_3 + (Am_1 + Bm_2 + Cm_3)x + (An_1 + Bn_2 + Cn_3)\eta = 0$$

And similarly any more complicated curve on the plate for which the equation can be found can be transferred to the plane of the orbit with far greater ease than by computing a series of positions point by point.

6. When a particular condensation in the matter emitted by a comet can be observed on a series of plates, it may be possible to determine the motion, and hence to infer the law of force which is acting. The difficulties in the way of identifying such a conden-

sation and measuring it with any approach to accuracy are very great. Hitherto attempts to do this have not met with very great success. But it is a problem of immense interest. If the equation of the orbit is

$$p = \rho\{1 + e\cos(\theta - \gamma)\}$$

and we have a series of positions (x_r, y_r) at times t_r, of which the number must be at least three, the constants will be given by the equations

$$p = \rho_r + ex_r\cos\gamma + ey_r\sin\gamma$$

for these will determine p, $e\cos\gamma$ and $e\sin\gamma$. When these have been found, three cases may arise according as (1) $e < 1$, (2) $e > 1$ and p is positive, (3) $e > 1$ and p is negative, these corresponding to motion on an ellipse, on the concave and on the convex branch of an hyperbola respectively.

7. It may be convenient to set down here the formulæ which suffice to determine the constant of force in each of the three cases.

(1)
$$\begin{aligned}
&x = \rho\cos\theta,\ y = \rho\sin\theta,\ v = \theta - \gamma\\
&e = \sin\phi,\ a = p\sec^2\phi\\
&\tan\tfrac{1}{2}\mathrm{E} = \tan(45^\circ - \tfrac{1}{2}\phi)\tan\tfrac{1}{2}v\\
&\mu^{\frac{1}{2}}a^{-\frac{3}{2}}(t - \mathrm{T}) = \mathrm{E} - e\sin\mathrm{E}\\
&\mu = k^2(1 - \mu')
\end{aligned}$$

which determines μ', the ratio of the repulsive action to gravitation, k being the Gaussian constant.

(2)
$$\begin{aligned}
&x = \rho\cos\theta,\ y = \rho\sin\theta,\ v = \theta - \gamma\\
&e = \sec\phi,\ a = p\cot^2\phi\\
&\tan\tfrac{1}{2}\mathrm{F} = \tan\tfrac{1}{2}\phi\tan\tfrac{1}{2}v\\
&\mu^{\frac{1}{2}}a^{-\frac{3}{2}}(t - \mathrm{T}) = e\tan\mathrm{F} - \mathrm{Log}\tan(45^\circ + \tfrac{1}{2}F)\\
&\mu = k^2(1 - \mu')
\end{aligned}$$

which determines μ' as before. Log denotes natural logarithm.

(3) For clearness, when p as determined in § 6 is negative, we simply change its sign and rewrite the equation of the convex branch

$$p = -\rho + e\rho\cos(\theta - \gamma - \pi)$$

and then

$$\begin{aligned}
&x = \rho\cos\theta,\ y = \rho\sin\theta,\ v = \theta - \gamma - \pi\\
&e = \sec\phi,\ a = p\cot^2\phi\\
&\tan\tfrac{1}{2}\mathrm{G} = \cot\tfrac{1}{2}\phi\tan\tfrac{1}{2}v\\
&\mu^{\frac{1}{2}}a^{-\frac{3}{2}}(t - \mathrm{T}) = \mathrm{Log}\tan(45^\circ + \tfrac{1}{2}\mathrm{G}) + e\tan\mathrm{G}\\
&\mu = k^2(\mu' - 1)
\end{aligned}$$

which again determines μ' in terms of gravitation. Astronomical units are assumed throughout.

8. If three observations of a condensation are available, it is evident that only a pair of times is required to determine μ and T, and that the time of the third observation serves as a control. A more satisfactory control must be looked for in a greater number

of observations, and the success with which they can be brought into harmony. If discrepancies present themselves, there are only too many causes which may contribute to produce them. These include not only false identifications and want of accuracy in a particularly difficult class of measurement, but also the possibly false assumption that the matter is moving in the plane of the orbit, and an error in the hypothesis that the repulsive force varies as the inverse square of the distance. It is to be hoped that those who have reaped such a brilliant success in the field of photographic observation will be undaunted by the patent difficulties, and will arrive at some more definite conclusions as to the laws of cometary forms than we can be said to have reached at the present time.

University Observatory, Oxford:
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The Clouds of Venus and their Significance. By Arthur W. Clayden, M.A., Principal of the Royal Albert Memorial University College, Exeter.

If it be assumed that the visible surface of Venus is the outer face of a heavily cloud-laden atmosphere, comparable in mass and composition with our own, it seems that certain deductions may be based on its observed features which have not hitherto been pointed out, though they convey valuable hints as to the planet's physical condition, the position of its axis, and the period of rotation.

There is no need to recapitulate here the many reasons for believing that the surface visible to most telescopes is actually cloud, and the sequel will show that such a view is not incompatible with the detection of sharp markings such as have been described by specially favoured observers. My own observations have been made with the 6·8 inch refractor by Tulley, described in vol. ii. of the Society's *Memoirs.* Its long focus (144 inches) gives excellent definition of Jupiter and Saturn, when the seeing is good, with powers up to 450; but it has never shown me anything on Venus which does not seem easily explicable on the assumption that the planet is shrouded in clouds, and that its atmosphere bears something like the same relation to its mass as our own atmosphere does to the Earth's.

Taking this for granted, let us consider how those clouds should be distributed according to different views as to the period of rotation and the position of the polar axis.

First let us assume that the axis makes a large angle with the plane of the orbit, and that the period of rotation is coincident with the revolution. That is to say, that the planet always turns the same face to the Sun—one side always exposed to the solar glare,

14